Underactuated approach for the control-based forward dynamic analysis of acquired gait motions

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ABSTRACT

This paper presents a method to carry out the analysis of acquired gait motions through a control-based forward dynamic approach. Unlike some well-established control-based methods that consider inputs for all the system degrees of freedom, the current work proposes the more realistic alternative of having inputs at joint level only, thus leading to an underactuated system. The ground reactions come from a foot-ground contact model which is built in a pre-processing stage. Different sets of outputs to be tracked by the joint controllers are evaluated. It is observed that choosing as outputs the weighted trajectories of all the system degrees of freedom yields satisfactory results, and that including the weighted ground reactions provides only marginal improvement.

Keywords: Biomechanics, Gait, Control-based forward dynamics, Foot-ground contact model, Underactuated system.

1 INTRODUCTION

The analysis of acquired gait motion through forward dynamics instead of traditional inverse dynamics offers certain advantages, such as superior dynamic consistency [1], ability to consider muscle activation and contraction dynamics when descending at muscular level [2], and feasible computation of contact forces between the subject and the ground or assistive devices [3]. Furthermore, since the forward dynamic analysis implies the integration in time of the model equations of motion, it must face the inherent challenges of gait dynamics (intermittent contact, stability, etc.), and can be perceived as an intermediate step to motion prediction, having less uncertainty as the resulting motion is known.

In a previous work [4], the authors addressed the forward dynamic analysis of an acquired gait motion by means of trajectory tracking controllers associated to all the degrees of freedom of the model. It was shown that the computed torque control (CTC) method provided good accuracy and was extremely robust with respect to the selected gain values. In fact, the computed muscle control (CMC) proposed in [2] at muscular level, uses the CTC method at joint level (introducing then an optimization loop to compute the muscular excitations that generate the obtained joint torques).

However, the human body is not a fully-actuated system, but an underactuated one. Therefore, if an approach closer to reality is sought, it is not admissible to control the six degrees of freedom of the base body (usually, the pelvis or the foot in contact with the ground). Instead, actuators can only be associated to human joints, while the external reactions coming through the feet can be represented by foot-ground contact models. An example of this approach can be found in [5] for a jumping exercise.

In this work, the forward dynamic analysis of an acquired gait motion is performed, considering the human body as an underactuated system and, hence, placing controlled actuators at the joints only. Controllers are supposed to track a number of outputs, which can be trajectories, forces or a combination of both. The objectives are to check whether the acquired motion can be approximated through this method, and to determine which is the specific implementation that best reproduces the motion, the joint drive torques and the ground reactions. The authors believe that gaining insight into this problem will be helpful for the more challenging topic of motion prediction.

The remaining of the paper is organized as follows. Section 2 presents the experiment data and the human model, including the foot-ground contact model. Section 3 describes the proposed method to carry out the control-based forward dynamic analysis of the acquired motion. Section 4 explains different alternatives of output choices and the corresponding results. Finally, the conclusions of the paper are drawn in Section 5.

2 EXPERIMENT AND MODEL

Gait data from a healthy adult male, 27 years old, mass 84 kg and height 1.75 m has been taken from the Library of Computational Benchmark Problems [6] developed by the IFToMM Technical Committee for Multibody Dynamics. The benchmark problem, named Gait 2D, provides the histories of the markers used to optically capture the motion of the mentioned subject, along with the ground reactions measurements provided by force plates. Furthermore, it gives the parameters defining the 12-segment, 14-degree-of-freedom planar human model shown in Figure 1 (right), used by the benchmark authors to perform an inverse dynamic analysis to derive the joint drive torques that generated the motion and the motion-consistent ground reactions (slightly different from the measured ones). The histories of these magnitudes, as well as the histories of the Cartesian coordinates and angles of the planar model in Figure 1, are also supplied.

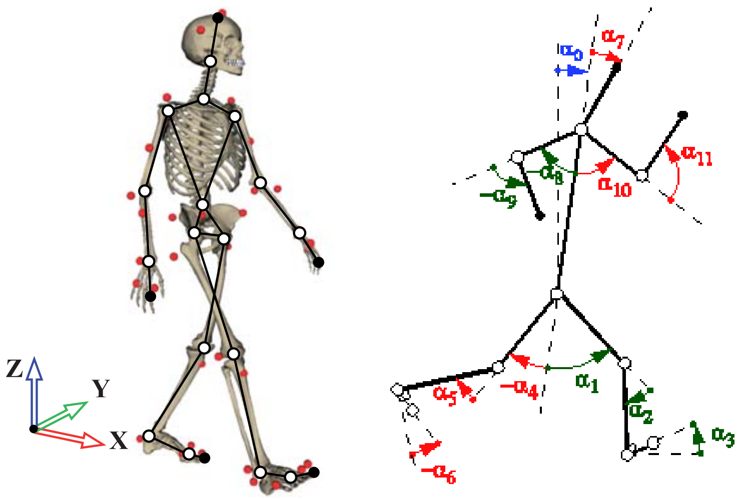

Figure 1. Original 3D subject model and 2D benchmark version.

For the purpose of conducting the forward dynamic simulation of the mentioned planar model, the state-space matrix-R formulation [7] has been applied,

$$\mathbf{M}\ddot{\mathbf{z}} = \mathbf{Q} + \mathbf{B}\mathbf{u} \tag{1}$$

where $\mathbf{M}$ is the system mass matrix, $\ddot{\mathbf{z}}$ is the vector of second time-derivatives of the coordinates (accelerations), $\mathbf{u}$ is the vector of actuations (less than the system degrees of freedom), projected to the coordinates space through matrix $\mathbf{B}$, and $\mathbf{Q}$ is the vector of velocity-dependent and remaining applied forces (gravitational and ground reactions).

The configuration vector $\mathbf{z}$ of 14 independent coordinates that has been selected in this work is formed by the two Cartesian coordinates of the hip and the angle between vertical axis and trunk (three degrees of freedom of the base body), along with the 11 relative angles illustrated in Figure 1 (right),

$$\mathbf{z}^{\mathrm{T}} = \{ x \quad y \quad \alpha_0 \quad \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4 \quad \alpha_5 \quad \alpha_6 \quad \alpha_7 \quad \alpha_8 \quad \alpha_9 \quad \alpha_{10} \quad \alpha_{11} \} \tag{2}$$

The equations of motion are integrated in time by means of the single-step implicit trapezoidal rule, with the accelerations $\ddot{\mathbf{z}}$ as primary variables.

The foot-ground contact model plays a key role in the proposed problem. If a force model is chosen, the system is certainly underactuated, and control methods for such types of systems must be used, with the additional difficulty of the unstable nature of gait. If a constraint method is selected seeking to have a fully-actuated system at all times, constraints must be alternatively imposed to the feet (thus perturbing the continuous motion they experience during gait, even at the stance phase), and the impact at landing must be dealt with in some way. Consequently, a force model has been used in this work.

Moreover, there is a problem that must be faced when using foot-ground force contact models in the forward dynamic analysis of acquired gait motions: the selection of the contact model parameters and, more importantly, of the feet boundaries. A not sufficiently good location of feet boundaries can yield huge contact forces that make the simulation fail. Therefore, an optimization method to select the mentioned characteristics of the contact model, similar to the one proposed in [8], is required to be applied prior to the forward dynamic analysis as a pre-processing stage, to ensure reasonable contact forces during the simulation.

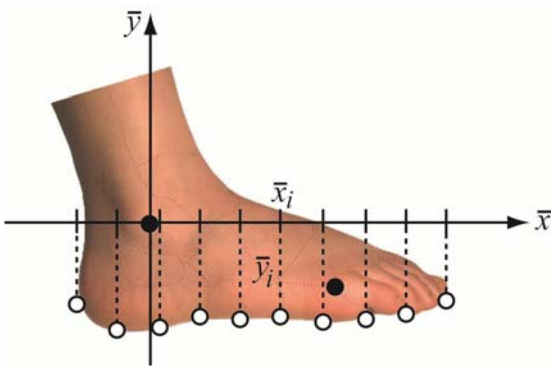

Figure 2. Foot boundary definition.

The nonlinear volumetric contact model proposed in [9] has been used, for which the normal and tangential contact forces are defined as,

$$\begin{aligned} f_n &= k_h V^h + a_h V v_{cn} \\ f_t &= -\mu(v_{ct}) f_n \quad ; \quad \mu(v_{ct}) = \mu_f \arctan(v_{ct}/v_s) \end{aligned} \tag{3}$$

where V is the interpenetration volume, k_h is the hyper-volumetric pseudo-stiffness, h is an exponent which depends on the volumetric stiffness and geometrical properties, a_h is the foundation stiffness multiplied by the damping, v_{cn} and v_{ct} are the normal and tangential velocities at the centroid of the deformed volume, respectively, μ is the friction coefficient, μ_f is the asymptotic friction coefficient, and v_s is a shape factor. The values of these parameters have been taken from [9], except for k_h and a_h, which have been reduced, since the values in [9] are for spheres and the model in this work is two-dimensional and, hence, it would assume a 1 m length in the direction orthogonal to the plane. The values of all the parameters are listed in Table 1.

Table 1. Parameters of the foot-ground contact model.

k_h	h	a_h	μ_f	v_s
10^5	0.79	$2\cdot10^6$	0.34	0.034

The optimization pre-process to set the feet boundaries is as follows. For each foot, a local reference system $(\bar{x},\bar{y})$ is defined as shown in Figure 2, with the origin at the ankle and the x-axis horizontal in the support position. Then, 10 equally-spaced points spanning the whole foot are taken along the x-axis. The $\bar{y}$-coordinates of these points, $\bar{y}_i$, $i=1,2,...,10$, which will serve to define the foot boundary through cubic splines, are the design variables of the optimization problem. The cost function to be minimized is the discrepancy between the histories of the ground reactions provided by the foot-ground contact model and those obtained from the inverse dynamic analysis. Normal and tangential forces, as well as the reaction moment, are considered into the cost function, scaling the reaction moment by a factor of 100 in order to balance the weight of the three components. The genetic algorithm *ga* from Matlab has been used, for which no initial guess is required, and the resulting feet boundaries are depicted in Figure 3.

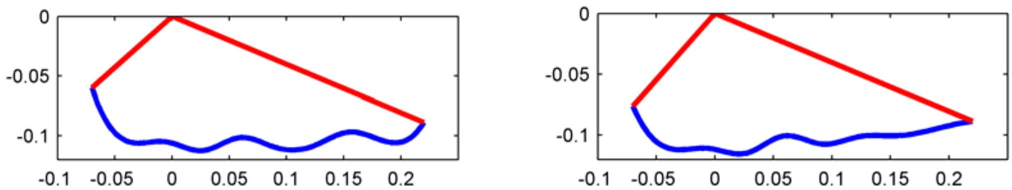

Figure 3. Feet boundaries obtained from the optimization pre-process.

3 CONTROL-BASED FORWARD DYNAMIC ANALYSIS

As said in the Introduction, the objective of this work is to perform the forward dynamic analysis of an acquired gait motion, by placing actuators at the joints only, thus leading to an underactuated system. Controllers governing the actuators are to track a number of outputs, which can be trajectories, forces or a combination of both. In what follows, a CTC-like approach for underactuated systems is described, which provides the inputs of the controllers as functions of the mentioned outputs.

The equations of motion of the underactuated system have been provided in (1), but are reproduced here for clarity,

$$\mathbf{M}\ddot{\mathbf{z}} = \mathbf{Q} + \mathbf{B}\mathbf{u} \tag{4}$$

The required outputs are considered to be either functions of the coordinates (e.g. joint trajectories), or of the coordinates and their first derivatives (e.g. ground reactions produced by a force model).

$$\mathbf{y} = \begin{Bmatrix} \mathbf{y}_1(\mathbf{z}) \\ \mathbf{y}_2(\mathbf{z},\dot{\mathbf{z}}) \end{Bmatrix} \tag{5}$$

Differentiating (5) with respect to time (twice for $\mathbf{y}_1$ and once for $\mathbf{y}_2$), and substituting then $\ddot{\mathbf{z}}$ from (1) yields,

$$\hat{\mathbf{y}} = \begin{Bmatrix} \ddot{\mathbf{y}}_1(\mathbf{z}) \\ \dot{\mathbf{y}}_2(\mathbf{z},\dot{\mathbf{z}}) \end{Bmatrix} = \begin{bmatrix} \dot{\mathbf{H}}_{1\mathbf{z}} & \mathbf{H}_{1\mathbf{z}} \\ \mathbf{H}_{2\mathbf{z}} & \mathbf{H}_{2\dot{\mathbf{z}}} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{z}} \\ \ddot{\mathbf{z}} \end{Bmatrix} = \begin{bmatrix} \dot{\mathbf{H}}_{1\mathbf{z}} \\ \mathbf{H}_{2\mathbf{z}} \end{bmatrix} \dot{\mathbf{z}} + \begin{bmatrix} \mathbf{H}_{1\mathbf{z}} \\ \mathbf{H}_{2\dot{\mathbf{z}}} \end{bmatrix} \ddot{\mathbf{z}} = \mathbf{A}\dot{\mathbf{z}} + \mathbf{D}\ddot{\mathbf{z}} = \mathbf{A}\dot{\mathbf{z}} + \mathbf{D}\mathbf{M}^{-1}\left(\mathbf{Q} + \mathbf{B}\mathbf{u}\right) \tag{6}$$

so that the vector of actuations **u** can be worked out from Eq. (6) as,

$$\mathbf{u} = \left(\mathbf{D}\mathbf{M}^{-1}\mathbf{B}\right)^{-1}\left(\hat{\mathbf{y}} - \mathbf{A}\dot{\mathbf{z}} - \mathbf{D}\mathbf{M}^{-1}\mathbf{Q}\right) \tag{7}$$

Now, calling $\mathbf{P} = \mathbf{D}\mathbf{M}^{-1}\mathbf{B}$, and considering that feedback is introduced for the outputs, it results,

$$\mathbf{u} = \mathbf{P}^{-1}\left(\begin{Bmatrix} \ddot{\mathbf{y}}_1^* + \mathbf{C}_\mathrm{v}\left(\dot{\mathbf{y}}_1^* - \dot{\mathbf{y}}_1\right) + \mathbf{C}_\mathrm{p}\left(\mathbf{y}_1^* - \mathbf{y}_1\right) \\ \dot{\mathbf{y}}_2^* + \mathbf{K}_\mathrm{p}\left(\mathbf{y}_2^* - \mathbf{y}_2\right) \end{Bmatrix} - \mathbf{A}\dot{\mathbf{z}} - \mathbf{D}\mathbf{M}^{-1}\mathbf{Q} \right) \tag{8}$$

where the asterisk indicates the desired values of the outputs, different from the current ones (without asterisk), and $\mathbf{C}_\mathrm{v}$, $\mathbf{C}_\mathrm{p}$ and $\mathbf{K}_\mathrm{p}$ are diagonal matrices containing the gains associated to each output.

If the number of outputs is equal to that of actuators, matrix **P** is square and the required inputs can be determined from (8). If the number of outputs is greater than that of actuators, the required outputs can be satisfied in a minimum squares sense only, the system of equations to be solved being,

$$\mathbf{u} = \left(\mathbf{P}^\mathrm{T}\mathbf{W}\mathbf{P}\right)^{-1}\mathbf{P}^\mathrm{T}\mathbf{W}\left(\begin{Bmatrix} \ddot{\mathbf{y}}_1^* + \mathbf{C}_\mathrm{v}\left(\dot{\mathbf{y}}_1^* - \dot{\mathbf{y}}_1\right) + \mathbf{C}_\mathrm{p}\left(\mathbf{y}_1^* - \mathbf{y}_1\right) \\ \dot{\mathbf{y}}_2^* + \mathbf{K}_\mathrm{p}\left(\mathbf{y}_2^* - \mathbf{y}_2\right) \end{Bmatrix} - \mathbf{A}\dot{\mathbf{z}} - \mathbf{D}\mathbf{M}^{-1}\mathbf{Q} \right) \tag{9}$$

with **W** the weight diagonal matrix which assigns more weight to more relevant outputs.

As the conventional CTC, this method is quite robust with respect to the gain values. In this work, $\mathbf{C}_\mathrm{p}$ has been given the value 10^3, $\mathbf{C}_\mathrm{v}$ the value corresponding to critical damping for the error dynamics [10], i.e. $\mathbf{C}_\mathrm{v} = 2\sqrt{\mathbf{C}_\mathrm{P}} = 2\sqrt{10^3} = 63.2456$, and $\mathbf{K}_\mathrm{p}$ the value 10^3.

4 OUTPUT SELECTION

Different alternatives in the choice of the outputs have been investigated, looking for the one that yields the best agreement between the acquired motion and the result of the forward dynamic simulation. Here, the most representative options are described. Note that the human model considered has 14 degrees of freedom and 11 inputs (joint actuators).

The first strategy tested (case 1) was to define as many outputs that inputs, i.e. 11, choosing as outputs some 11 coordinates from the configuration vector **z** defined in (2). The selected outputs were the joint relative angles, while the three coordinates of the base body (trunk) were left free. The upper part of (8) was used to calculate the required inputs along the simulation.

The second strategy tested (case 2) was to define more outputs than inputs, choosing as outputs the 14 coordinates of the configuration vector **z** defined in (2). The upper part of (9) was used to calculate the required inputs along the simulation. In this case, the weights for the outputs must be decided, and used to build the weight matrix **W**. The evolutionary optimization method known as *Covariance Matrix Adaptation Evolution Strategy* (CMA-ES) [11], whose Matlab implementation has been downloaded from https://www.lri.fr/~hansen/, was applied to find the optimum values of the weights, which are gathered in Table 2 (rounded).

Table 2. Weights of the different outputs in case 2.

x	y	α_0	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}	α_{11}
98	98	99	8	40	0.01	2	27	16	30	2	42	2	98

The third strategy tested (case 3) was again to define more outputs than inputs, but choosing this time as outputs the 14 coordinates of the configuration vector **z** defined in (2) plus the three ground reaction components (normal and tangential forces and reaction moment) at each foot, i.e. 6 ground reaction components, leading to a total number of 20 outputs. The whole equation (9) was used to calculate the required inputs along the simulation. In this case, the weights required to build matrix **W** must be decided too: the kinematic outputs have been assigned a weight value 1, the ground normal reaction forces have been assigned a weight value 10^{-3}, and both the ground tangential reaction forces and the reaction moments have been assigned a weight value of 10^{-2}, so that the 20 outputs have a similar order of magnitude.

The agreement between the acquired motion (reference) and the result of the forward dynamic simulation (case 1, 2 or 3) is measured by means of the RMSE between the histories of the Cartesian coordinates of the black/white points in the human model shown in Figure 1 (right) corresponding to the reference and the case considered (1, 2 or 3). The obtained RMSE values are listed in Table 3.

Table 3. RMSE of the resulting motion with respect to the acquired motion.

Case 1	Case 2	Case 3
0.1070	0.0382	0.4818

It can be seen that the best correlation is obtained for case 2, i.e. when all the independent coordinates in the configuration vector **z** are selected as outputs, although in a minimum squares sense. Note also that, in this case, the weights of the different outputs have been optimized.

More detailed results are presented in the following. Figure 4 shows the histories of the three coordinates of the base body (trunk) for the three cases studied, and compares them with the result of the inverse dynamic analysis, taken as reference.

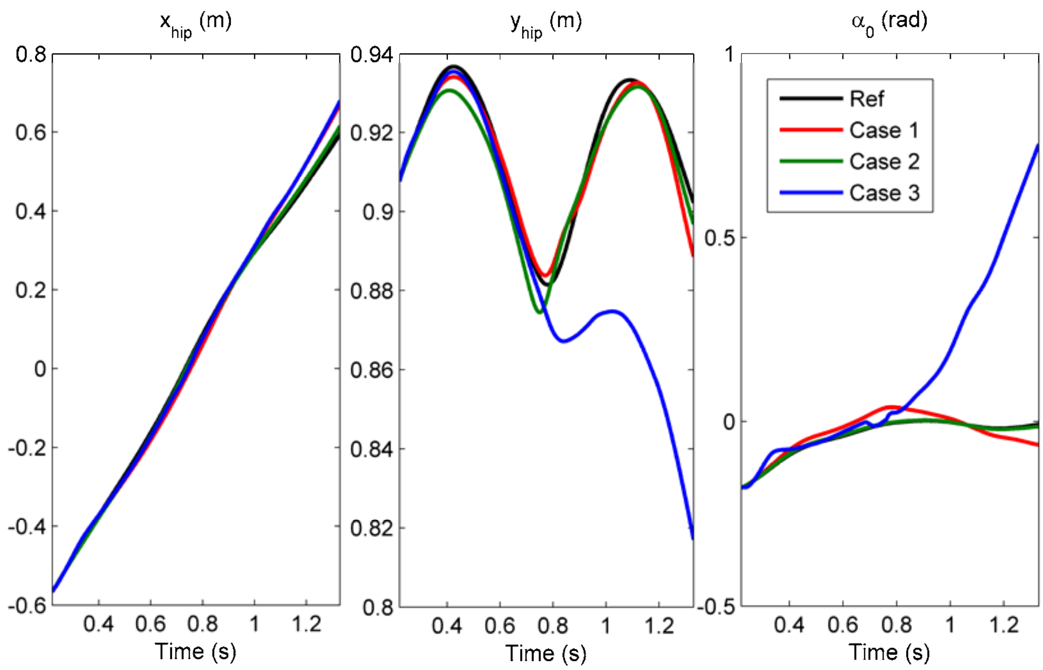

Figure 4. Coordinates of the base body (trunk) obtained with different control strategies vs reference.

Figure 5 plots the histories of the right hip angle, α_1, for the three cases studied, and compares them with the result of the inverse dynamic analysis, taken as reference.

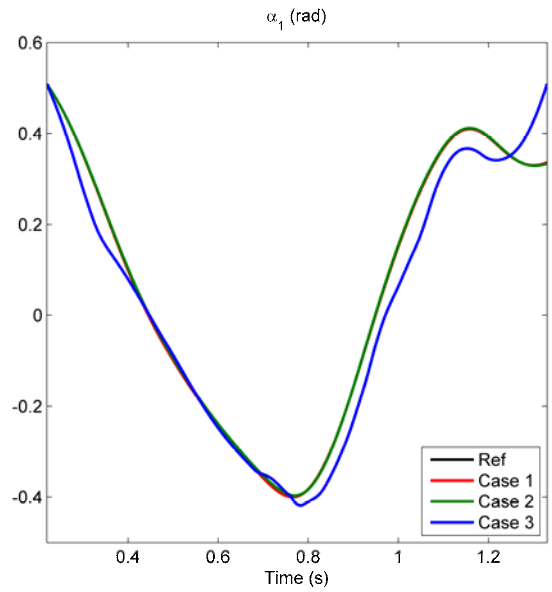

Figure 5. Right hip angle obtained with different control strategies vs reference.

Figure 6 gathers the histories of the normal ground reaction force at the left foot for the three cases studied, and compares them with the result of the inverse dynamic analysis, taken as reference. Note that the simulation starts with the heel-strike of the right foot.

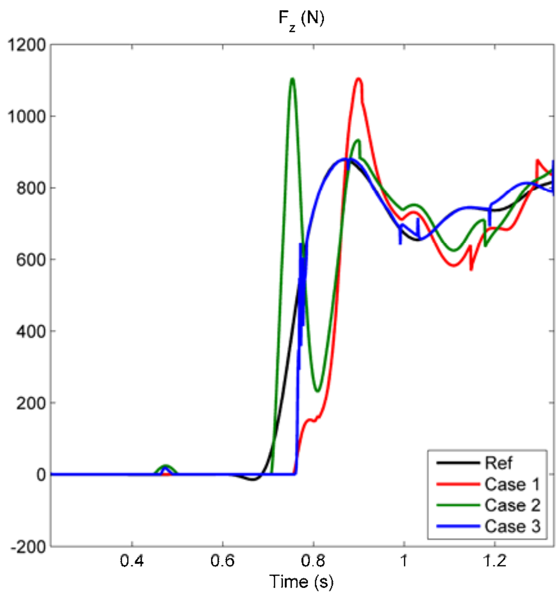

Figure 6. Normal ground reaction force obtained with different control strategies vs reference.

In the last three figures, it can be seen that cases 1 and 2 provide good motion correlation, although some peaks can be observed in the ground reactions. Conversely, case 3 yields an excellent correlation of the ground reactions, at the prize of being far from following the motion. As said before, weighting factors have been assigned in this case so as to provide equivalent relevance to all the outputs. Perhaps reducing the weight of ground reactions could lead to a better fitting of the motion.

To provide a clearer illustration of the obtained gaits, Figure 7 compares the resulting model motion with the acquired motion in the second case (best correlation).

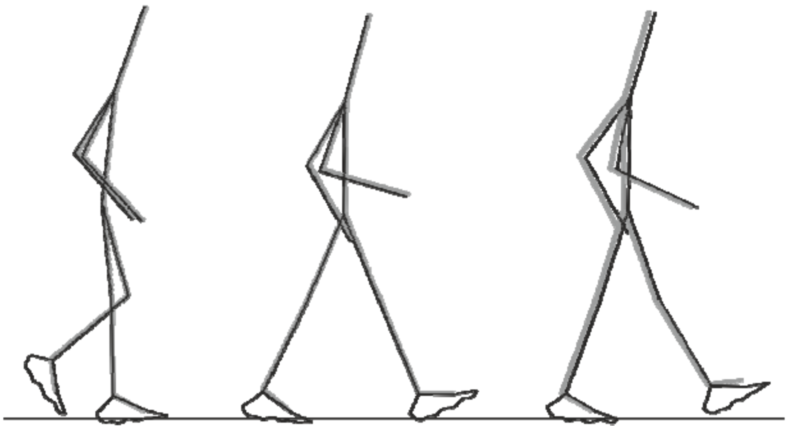

Figure 7. Model motion in the second case vs reference.

As it can be seen in Figure 7, the main source of discrepancy in case 2 is the sliding between foot and ground during the support phase, due to the tangential force model. A stick-slip model could help to improve this aspect.

5 CONCLUSIONS

An acquired gait motion has been analyzed by a control-based forward dynamic approach, considering actuation at the joints only, so that the resulting system is underactuated. A CTC-like method has been applied to obtain the control inputs. Three cases have been tested: selecting as outputs as many system degrees of freedom as inputs; selecting as outputs all the system degrees of freedom in a minimum squares sense; selecting as outputs all the system degrees of freedom plus the ground reactions in both feet, again in a minimum squares sense.

First of all, it has been demonstrated that the definition of feet boundaries which are in good agreement with the ground reactions is essential to preserve gait stability during the simulation.

Regarding the three cases tested, the obtained results show, on the one hand, that tracking the motion through the proposed control-based forward dynamic approach is possible, despite the unstable character of gait. On the other hand, it is observed that imposing the motion of all the system degrees of freedom, yet in a minimum squares sense (case 2), works better than exactly imposing the motion of as many degrees of freedom as actuators (case 1). Moreover, the former strategy is quite robust with respect to the values of the weighting factors. If, besides the motion of all the system degrees of freedom, the ground reactions are also included as outputs (case 3), the correlation deteriorates.

As described in the paper, only a limited number of tests have been carried out. Therefore, the conclusions extracted here cannot be considered definitive, since many options can still be explored within the implemented strategies. For example, in case 1, a criterion could be applied consisting of selecting as outputs those coordinates corresponding to the largest pivots of matrix **P**, since the remaining coordinates are those with the highest capacity to control the system motion through their variation. In case 3, instead of assigning equal weight values to all the outputs, optimized sets of weighting values could be sought.

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