MOBILE ROBOT LOCALIZATION. REVISITING THE TRIANGULATION METHODS

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Abstract: Localization is one of the fundamental problems in mobile robot navigation. In this context, triangulation is used to determine the robot pose from landmarks position and angular measurements. The method based on *circle intersection* is the preferred one because it is independent of the robot heading. Nevertheless, it becomes undetermined when the robot point is on the circumference that contains the landmarks used. To cope with it, a triangulation method based on *straight lines intersection* is presented in this paper. The robot heading angle, which is needed in this method, can be accurately determined by means of a geometrical procedure. The accuracy of the presented approach is evaluated and compared to that of alternative methods by means of experimental results and computer simulations.

Keywords: Automated guided vehicles, Mobile robots, Positioning systems, Robot navigation, Kinematics.

1. INTRODUCTION

Mobile robots are increasingly used in flexible manufacturing industry and service environments. The main advantage of these vehicles is that they can operate autonomously in their workspace. To achieve this automation, mobile robots must include a localization –or positioning– system in order to estimate the robot pose –position and orientation– as accurately as possible (Leondes, 2000). In the past two decades, a number of different approaches have been proposed to solve the localization problem. These can be classified into two general groups (Borenstein, *et al*., 1997): relative and absolute localization methods.

In relative localization, dead reckoning methods –odometry and inertial navigation– are used to calculate the robot position and orientation from a known initial pose. Odometry is a widely used localization method because of its low cost, high updating rate, and reasonable short path accuracy. However, its unbounded growth of time integration errors with the distance travelled by the robot is unavoidable and represents a significant inconvenience (Kelly, 2004). Several approaches have been done to cope with the odometry error propagation (Wang, 1988; Tonouchi, *et al*., 1994).

Conversely, absolute localization methods estimate the robot position and orientation by detecting distinct features of a known environment. Most of the work published integrates a prediction phase, based on the odometric data and the robot kinematics, and a correction –or estimation– phase that takes into account external measurements. The most used methods are based on Kalman filtering (Leonard and Durrant-Whyte, 1991; Hu and Gu, 2000; Jensfelt and Christensen, 2001) and Bayesian localization (Burgard, *et al*., 1996; Dellaert, *et al*., 1999). Bayesian localization methods are robust to complex, dynamic and badly mapped environments, but are in general less accurate. Finally, other authors deal with absolute localization using bounded-error state estimation applying interval analysis such in (Kieffer, *et al*., 2000).

The presented localization method uses an extended Kalman filter (EKF) to estimate at any time the angles –relative to the robot frame– of the straight lines from the sensor –a rotating laser scanner in this case– to the landmarks used (Font and Batlle, 2005). Once the angles are estimated, the presented triangulation method based on *straight lines intersection* is used to determine the robot pose.

Several triangulation methods have been presented to determine the robot pose. These methods refer to any process which solves a system of simultaneous algebraic or transcendental equations –whether or not they are reducible to an equivalent problem involving triangles– (Kelly, 2003). In mobile robotics, triangulation occurs often in the context of artificial landmarks. However, any natural aspects of the environment –such as walls or corners– whose positions are known, and are detected by a sensor whose indications depend on their position relative to the sensor, establish a triangulation context.

In section 2, triangulation methods for mobile robot localization and their properties are described. The angular state estimator used is also explained in this section. Next, in section 3, the proposed triangulation approach based on straight lines is presented. Finally, in sections 4 and 5, computer simulations and experimental results showing the positioning accuracy of the presented method are reported.

2. TRIANGULATION BASED MOBILE ROBOT LOCALIZATION

By means of angular triangulation it is possible to determine the robot position and orientation from the landmarks position and the measured angles θ_1, θ_2 and θ_3 –relative to the robot longitudinal axis– for three of them (McGillem and Rappaport, 1989; Cohen and Koss, 1992) as it can be seen in Fig. 1. As the accuracy of triangulation algorithms depends upon the point of observation and the landmark arrangements, more than three landmarks can also be used to improve accuracy (Betke and Gurvits, 1997).

In this paper, a laser positioning system has been considered. The main advantage of this system in industrial applications is its high positioning accuracy which is required in certain practical situations such as loading, unloading or narrow crossings. The system consists of a rotating laser scanner and a group of catadioptric landmarks strategically placed.

The scanner emits a rotating laser beam that horizontally sweeps the environment and reflects back when it detects a landmark L_i. The system measures the angle θ_i of the reflected beam, relative to the vehicle longitudinal axis (Fig. 1), by means of an incremental encoder. For the consistent use of triangulation, the angles θ_i to each landmark must be known at the same mobile robot pose. This condition is directly fulfilled under robot static condition.

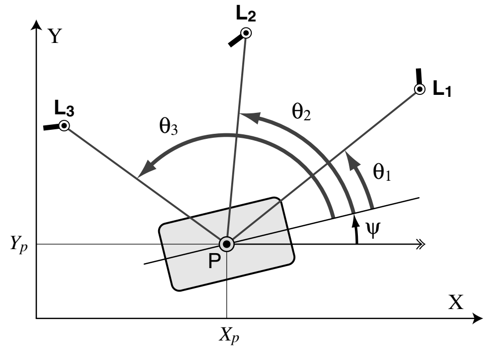


Fig. 1. Mobile robot position and orientation can be calculated from landmarks position and angles θ_1, θ_2 and θ_3 by triangulation.

However, under robot dynamic condition –robot in motion– triangulation cannot be directly applied because each of the landmarks is detected at a different robot pose (Batlle, *et al.*, 2004). The following subsection explains the proposed algorithm to solve this problem.

2.1 Angular state EKF used to allow triangulation under robot dynamic condition

To cope with triangulation under dynamic condition, an extended Kalman filter is used to estimate at any time the relative angles θ_i, according to the odometric evolution of these angles and the laser angular measurements. This method, presented in (Font and Batlle, 2005), allows the kinematically consistent use of triangulation methods. The angular state vector used in the proposed EKF is $\boldsymbol{x}_k = \{\theta_{1,k}, \theta_{2,k}, ..., \theta_{N,k}\}^T$, where N represents the number of viewed landmarks (in this case $N = 3$). The state transition function of the proposed EKF describes how the state $\boldsymbol{x}_k$ changes with time in response to the robot odometric measures $\boldsymbol{u}_k$ and a noise disturbance $\boldsymbol{w}_k$:

$$\boldsymbol{x}_k = \boldsymbol{f}\left(\boldsymbol{x}_{k-1}, \boldsymbol{u}_{k-1}, \boldsymbol{w}_{k-1}\right). \qquad (1)$$

The ith file (i = 1, ..., N) of this transition function $\boldsymbol{f}$, –associated to the evolution of angle θ_i– is obtained from the discrete time integration of the time evolution of angle θ_i, which can be expressed as:

$$\dot{\theta}_i = \frac{v_L \sin\theta_i - v_T \cos\theta_i}{\rho_i} - \dot{\psi}\ , \qquad (2)$$

where variable ρ_i stands for the distance between the scanner center P and landmark L_i, v_L and v_T are the longitudinal and transversal components of the velocity of P, and $\dot{\psi}$ represents the time evolution of the robot orientation angle (Fig. 2).

In this approach a forklift type mobile robot has been considered (Fig. 2). The kinematical expressions for v_L, v_T and $\dot{\psi}$ with respect to v and γ –velocity of the driving wheel center and steering angle respectively– which are measured by odometric sensors can be found in (Font and Batlle, 2005).

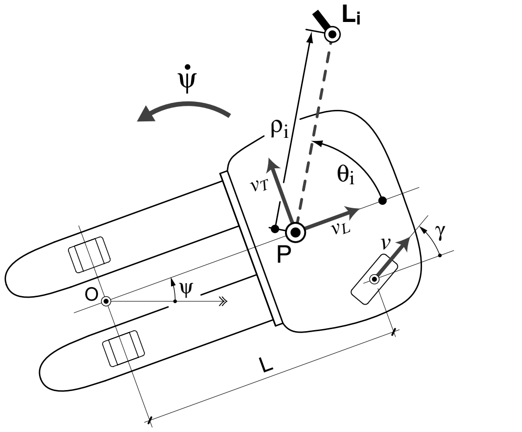


Fig. 2. Geometric and kinematical parameters considered in equation (2).

The measurement model of the EKF relates the state with the external measure $\boldsymbol{z}_k$ by means of the observation function $\boldsymbol{h}$:

$$\boldsymbol{z}_k = \boldsymbol{h}\left(\boldsymbol{x}_k, v_\theta\right), \tag{3}$$

where $v_\theta \sim N(0,\ \sigma_\theta^2)$ denotes the measurement noise associated to the laser sensor. Note that the external measures $\boldsymbol{z}_k$ are directly the state of the filter. This fact simplifies the required calculations and reduces the error associated with the linearization of $\boldsymbol{h}$.

2.2 Triangulation methods for robot localization

Once the relative angles θ_1, θ_2 and θ_3 are optimally estimated at any time, a triangulation method can be used to determine the robot pose. The one based on *circle intersection* seems to be the best in terms of robustness and computation time (Cohen and Koss, 1992). It determines the robot position as the intersection of two circumferences (McGillem and Rappaport, 1989), as it can be seen in Fig. 3.

These circumferences are determined from relative angles α and β between landmarks, which are obtained from θ_1, θ_2 and θ_3. The main advantage of this method is that it does not depend on the robot orientation. However, it has the drawback of becoming undetermined when the laser center P is on the circumference that contains the landmarks used.

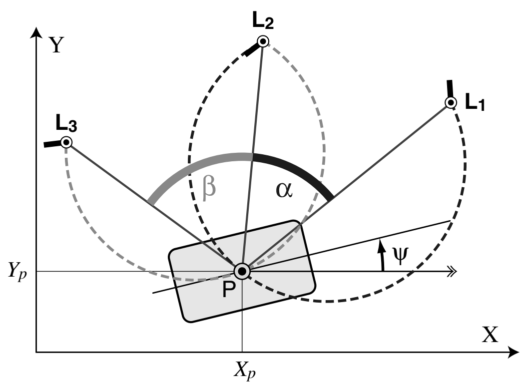


Fig. 3. Triangulation method based on circle intersection.

If the robot orientation angle ψ is accurately known, triangulation based on *straight lines intersection* can be carried out because the angles of the straight lines from point P to each landmark L_i are $\theta_i + \psi$ (i = 1, 2, 3). In fact, only two landmarks would be necessary provided that P is not aligned with them. The use of three unaligned landmarks avoids position indetermination regardless of the robot position.

However, if the robot orientation angle is not known, triangulation based on straight lines intersection cannot be directly applied. Cohen and Koss (1992) propose an iterative method that searches for the correct robot orientation within the interval (-90º, 90º) before triangulating. Its main drawback is that timing and precision are factors in the algorithm. If more accuracy is needed, more iterations are required to find the correct orientation, and then the running time of the algorithm increases.

3. PROPOSED TRIANGULATION METHOD BASED ON STRAIGHT LINES INTERSECTION

In this paper, a geometrical method that determines the robot correct orientation without performing any iteration is presented. Once the orientation is estimated, triangulation based on straight lines intersection can be performed.

The method starts by intersecting the straight lines from each landmark L_i with angle θ_i relative to the X axis. If the robot orientation is different from 0º or 180º the three determined points (O_{12}, O_{13} and O_{23}) are scattered and define a triangle, which is defined as the *orientation triangle* (Fig. 4).

It is obvious that a generic vertex O_{ij} of the orientation triangle belongs to the circumference that contains P, L_i and L_j (Fig. 5). In Fig. 5 C_{ij} represents the center of the defined circumference, which can also be easily determined because the positions of O_{ij}, L_i and L_j are known. Taking into account the geometry implied, it can be seen that the orientation triangle and the triangle with vertexes C_{12}, C_{13} and C_{23} –called the *centers triangle*– are similar because both of them have the same three angles (Fig. 6).

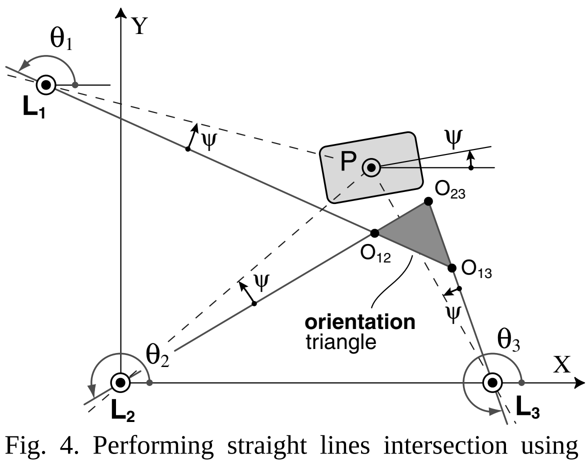


Fig. 4. Performing straight lines intersection using angles θ_i as line absolute angles generates the *orientation triangle* (vertexes O_{12}, O_{13}, O_{23}).

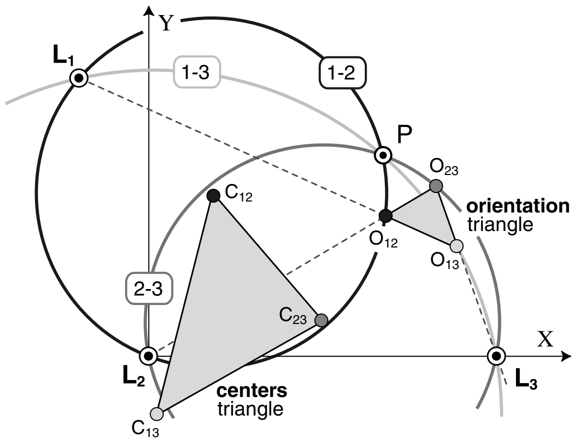


Fig. 5. Centers triangle and orientation triangle.

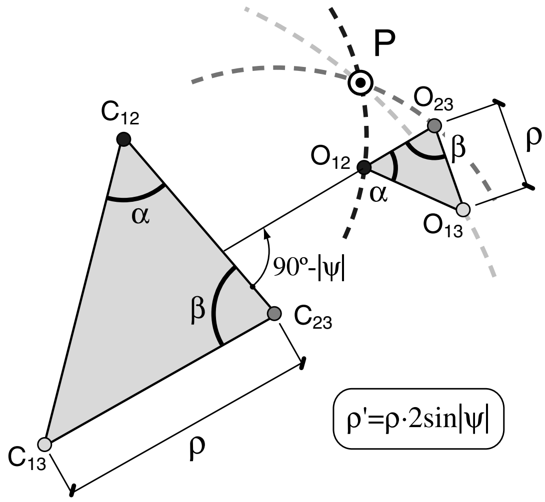


Fig. 6. Centers triangle and orientation triangle are similar because both have the same three angles.

It is well known that between two similar triangles there is a similarity ratio r that indicates the scale factor between two corresponding sides of the triangles. In the considered problem, it can be found that this ratio depends upon the robot orientation angle ψ. Taking into account the geometry implied, it can be demonstrated that the scale factor r between two corresponding sides of both triangles (see Fig. 6) can be expressed as:

$$r = \frac{\left\| \overline{\mathrm{O}_{ij}\mathrm{O}_{ik}} \right\|}{\left\| \overline{\mathrm{C}_{ij}\mathrm{C}_{ik}} \right\|} = 2\sin|\psi| . \tag{4}$$

It must be remarked that, if one vertex of the orientation triangle cannot be determined because P is aligned with two of the landmarks, then the similarity ratio r must be calculated by using the lengths of the two finite sides of the triangles. Once, this ratio r is known, the orientation estimation in absolute value can be readily obtained using expression (4).

The sign of ψ can be determined by means of the orientation angle of one side of the orientation triangle with respect to its corresponding side in the centers triangle: 90º–$|\psi|$ counterclockwise if $\psi > 0$ (Fig. 6); or 90º–$|\psi|$ clockwise if $\psi < 0$. Therefore, the sign of ψ coincides with the sign of the Z component of the vectorial product of one side of the centers triangle with the corresponding side of the orientation triangle:

$$\operatorname{sign}(\psi) = \operatorname{sign}\left[\overline{\mathrm{C}_{ij}\mathrm{C}_{ik}} \wedge \overline{\mathrm{O}_{ij}\mathrm{O}_{ik}}\right]_{\mathrm{Z}} . \tag{5}$$

The method above only determines angles within the interval (-90º, 90º). However, if the real orientation angle ψ of the vehicle is within the interval (90º, 270º), the method can also be used and the solution reached by means of it would be $\tilde{\psi} = \psi - 180^{\circ}$.

To avoid this indetermination in the robot heading angle, information of the robot pose estimation at the previous time step k–1 can be used. The difference between the estimated orientation by means of the geometrical method $\tilde{\psi}_k$, and the previous robot orientation ψ_{k-1} can not be greater than a threshold ε that depends on the robot kinematics and the time between updates. Then,

$$\begin{array}{ll} \textbf{if} & \left|\tilde{\psi}_k - \psi_{k-1}\right| > \varepsilon \rightarrow \psi_k = \tilde{\psi}_k + 180^{\circ} \\ \textbf{elseif} & \left|\tilde{\psi}_k - \psi_{k-1}\right| \le \varepsilon \rightarrow \psi_k = \tilde{\psi}_k \end{array} . \tag{6}$$

Once the orientation of the vehicle at the actual time step k is known, triangulation using straight lines intersection can be applied. As there is a positioning error related to the laser encoder resolution, the three positions associated to each pair of landmarks (1-2, 1-3 and 2-3) are calculated: P_{12}, P_{13}, P_{23}. Then, each position estimation P_{ij} is weighted with a factor W_{ij} that is inversely proportional to the maximum error that can be committed using landmarks L_i and L_j taking into account the laser scanner measurement resolution:

$$W_{ij} = \sin\alpha_{ij} \big/ \max\left(L_{ij}, 2\delta_{ij}\right) , \tag{7}$$

where $\alpha_{ij} = \theta_j - \theta_i$, is the bearing angle between landmarks L_i and L_j, L_{ij} is the distance between these landmarks, an δ_{ij} is the distance from the middle point of the segment $\overline{L_i L_j}$ to the observation point.

It is important to note that the geometrical method used to determine the robot orientation angle is not valid when point P is on the circumference that contains the three landmarks, because in this case both triangles used reduce to a point. Under this transitory circumstance or close to it, the presented method relies on robot odometry to estimate the orientation angle ψ:

$$\psi_k = \psi_{k-1} + \left[\frac{v_{k-1}}{L}\sin\gamma_{k-1}\right]\left(t_k - t_{k-1}\right) , \tag{8}$$

where variables v, γ and distance L are defined in Fig. 2. Once the orientation is odometrically estimated, the weighted straight lines intersection can be applied to determine robot position.

4. COMPUTER SIMULATIONS

Some computer simulations have been carried out to illustrate the accuracy of the presented localization method. A realistic model of a forklift mobile robot with a tricycle kinematics and the sensors used has been created with *Simulink 6.1* (included in MATLAB). By means of this model it is possible to check different localization methods under the same environmental and sensor noise conditions.

The simulated robot is provided with a laser scanner that rotates at 8 *Hz* and delivers an accuracy of 0.1 *mrad*, and the driving and steering encoders to determine the velocity of the driving wheel and its steering angle. From these odometric measurements, variables v_L, v_T and $\dot{\psi}$ used in the angular state EKF can be determined using the kinematical equations presented in (Font and Batlle, 2005).

During the simulation the center of the driving-steering wheel follows a 9.2 *m* trajectory at 0.5 ms^{-1}. This trajectory and the landmark layout are shown in Fig. 7. Only three landmarks are seen and the scanner center P is forced to cross twice the circumference that contains the landmarks. Under these conditions, triangulation based on *circle intersection* is not advisable because the positioning error greatly increases near this circumference and the method is undetermined over it.

The presented approach is also undetermined over this circumference because the robot orientation can not be determined. However, the odometric estimation of ψ close to this circumference allows the use of triangulation based on *straight lines*, which has no undetermined positions if three unaligned landmarks are used.

Table 1 show the statistical parameters of the lateral error e_{lat} between the actual robot trajectory and the calculated one when using the presented approach and triangulation based on *circle intersection*. The first column indicates the root mean square error, while the others indicate the mean and the standard deviation of the e_{lat} absolute value respectively. It can be noticed that, under the same conditions, the proposed method performs the best accuracy.

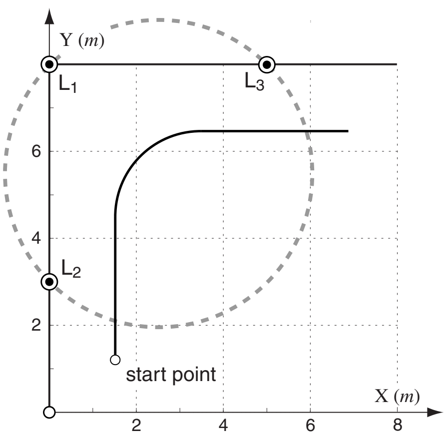


Fig. 7. Simulated trajectory and landmark layout.

Table 1 Statistical parameters of the lateral error using different triangulation methods

Localization method	**RMSE** e_{lat} [mm]	**mean** $\lvert e_{lat}\rvert$ [mm]	**std dev** $\lvert e_{lat}\rvert$ [mm]
presented method	2.5	1.9	1.6
circle intersection	4.6	3.3	3.2

5. EXPERIMENTAL RESULTS

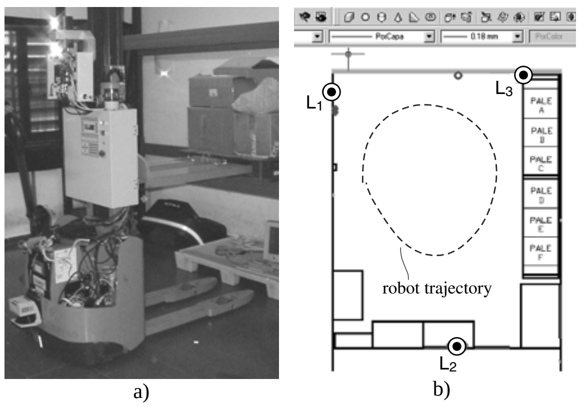
The method has also been tested on a real forklift mobile robot for industrial applications (Fig. 8a). The hardware used to support the method is an industrial PC (PC104 based) Pentium III clocked at 400 *MHz* on the robot. This PC runs with a real-time operative system RT-Linux 3.2. For the odometric and laser signals capture, specific firmware implemented by FPGA is applied.

The robot navigates through the laboratory shown in Fig. 8b, and the same three landmarks (positioned with sub-millimeter accuracy) have been used for all the configurations. In the experiments the robot has been manually guided with a velocity of the driving wheel center in the range of 0.14 – 0.18 ms^{-1}.

To validate the accuracy of the presented method, some points of the actual robot trajectory have been measured using a photogrammetric method based on a high resolution camera (Escoda, *et al*., 2005). The lateral error e_{lat} between the calculated and the actual points is taken as a measure of the accuracy, and both methods considered in section 4 are compared under realistic laboratory conditions. Table 2 illustrates the accuracy obtained by means of these methods.

Table 2 Statistical parameters of the lateral error during the experimental validation

Localization method	**RMSE** e_{lat} [mm]	**mean** $\lvert e_{lat}\rvert$ [mm]	**std dev** $\lvert e_{lat}\rvert$ [mm]
presented method	3.1	2.4	2.0
circle intersection	3.8	3.0	2.4



a) b)

Fig. 8. a) Forklift mobile robot used. b) CAD map of the environment and robot trajectory performed.

It can be seen from Table 2 that the presented method achieves better results than the one based on circle intersection. This is because it performs better near the circumference that contains the three landmarks.

During the experimental validation, a maximum lateral error of 7 *mm* –between the actual and the calculated points– has been obtained when using the presented method, while 9.3 *mm* has been the maximum error using triangulation based on circle intersection.

6. CONCLUSIONS

In this paper, a triangulation method based on straight lines intersection has been presented. This method accurately determines the required robot orientation angle using a geometrical procedure. To allow the kinematically consistent use of triangulation under robot dynamic condition –robot in motion–, an extended Kalman filter is used to estimate at each time step the angles –relative to robot longitudinal axis– of the straight lines from a laser sensor to each landmark.

An advantage of the presented approach is that the indetermination of robot position when the laser scanner center P is on the circumference that contains the three landmarks used, can be conveniently solved. As indetermination in this case is associated with orientation indetermination, it can be solved using the odometric estimation of the robot orientation when P is close to this circumference.

The presented approach has been compared to the usual triangulation method –based on circle intersection– by means of computer simulations and by means of experiments using a real forklift prototype. In both, computer simulations and real experiments, the presented method has turned out to be the best in terms of localization accuracy.

In the future, this method can be improved by using more than three landmarks.

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