

Two Approaches for the Dynamic Analysis of Impact in Biomechanical Systems

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Abstract Two main approaches are used when studying impact problems involving rigid bodies: impulsive and compliant. In an *impulsive approach*, the impact time interval is considered to be negligible, and so the system configuration is assumed to be constant. The final mechanical state can be obtained directly from the initial one by means of algebraic equations and energy dissipation assumptions. In a *compliant approach*, the colliding surfaces are modeled through nonlinear springs and dampers, and the differential equations of motion are integrated to solve the forward dynamics. In this paper, the performance of the two approaches is compared in two biomechanical application examples.

1 Introduction

The analysis of the impact dynamics is a major subject in biomechanics, since this phenomenon is present when the human body interacts with the environment. There are two main approaches for the dynamic analysis of impact: impulsive and compliant. The use of one or the other depends on the purpose of the study. There are no studies in the literature comparing the above-mentioned approaches when applied to biomechanical systems. The aim of this paper is to compare the performance of impulsive and compliant formulations by means of two application examples.

Impulsive approaches assume that the contact interaction is instantaneous and, therefore, the system configuration is constant. The integrated version of the equations of motion can be used to algebraically solve the forward dynamics of the sys-

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tem under nonsliding conditions. This approach is very useful to obtain performance indicators of the impact, such as the mechanical energy loss or the magnitude of contact impulses [1], with low computational cost.

In compliant approaches, the dynamics of contact interaction is solved continuously in time and the configuration is allowed to change. When contact is detected, the generated forces are added to the differential equations of motion. This analysis requires explicit models giving the variation of these forces as a function of the system state [2]. Compared to impulsive models, the use of compliant formulations results in a computationally costly way to solve the forward dynamics, since it requires the integration of the dynamics equations. Nevertheless, this description allows the estimation of forces at any time during the impact.

In this work, we compare the use of the two approaches to analyze situations where constraints are suddenly imposed. The two application examples are the heel-strike impact of a passive dynamic compass-gait biped (Fig. 1a), and the impact of the crutch with the ground in crutch locomotion (Fig. 1b). The energy loss at impact, and the post-impact velocities will be used as indicators to compare the performance of both formulations from the same pre-impact states.

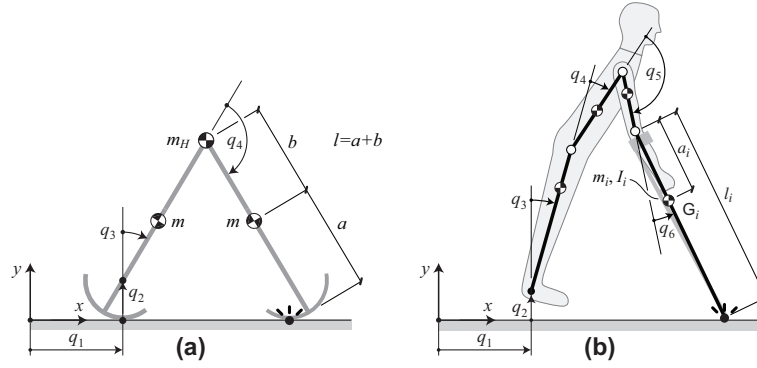


Fig. 1 Application examples: (a) Compass-gait biped. (b) Subject walking with crutches.

2 Contact Approaches for Impact Analysis

Impulsive Formulation. The situation of sudden addition of constraints can be characterized with bilateral impulsive constraints [1]. For a system described through n independent generalized velocities arrayed in vector $\dot{\mathbf{q}}$, these can be written as $\mathbf{A}\dot{\mathbf{q}}^+ = \mathbf{0}$, where $\dot{\mathbf{q}}^+$ is the value of $\dot{\mathbf{q}}$ at the post-impact instant, and \mathbf{A} is the Jacobian matrix of the constraints.

Impulsive approaches consider the impact interval to be very short in the characteristic time scale of the finite motion of the system. Therefore, the configuration is assumed constant. The time integration of the equations of motion over the impact interval $[t_i^-, t_i^+]$ leads to the following equations at the impulse-momentum level

$$\mathbf{M}(\dot{\mathbf{q}}^+ - \dot{\mathbf{q}}^-) = \mathbf{A}^T \bar{\boldsymbol{\lambda}}, \quad (1)$$

where \mathbf{M} is the mass matrix and $\bar{\lambda}$ is the vector containing the impulse of the constraint forces generated at impact. The post-impact generalized velocities $\dot{\mathbf{q}}^+$, the mechanical energy loss, and the constraint impulses $\bar{\lambda}$ can be determined as explained in [1]. This is based on the two projector operators \mathbf{P}_c and \mathbf{P}_a associated with the *constrained* and *admissible* motions of the mechanical system [3].

Compliant Formulation. Compliant approaches require the knowledge of the evolution of contact forces during the interval $[t_i^-, t_i^+]$. In this work, the normal contact force F_n is defined using a Hunt-Crossley model [2]

$$F_n = k_n (\delta_n)^{3/2} + \chi (\delta_n)^{3/2} \dot{\delta}_n, \quad (2)$$

where k_n is the normal stiffness according to the Hertz theory [4], and depends on the material properties and the surface curvature, δ_n (> 0) and $\dot{\delta}_n$ are the normal indentation between bodies and its time derivative, and χ is the hysteresis damping factor. Regarding the tangential force \mathbf{F}_t , we use a Coulomb's dry friction model that also accounts for the compliance of materials in the tangential direction. The compliant formulation of this force assuming no slip in the contact area is

$$\mathbf{F}_t = -k_t (\delta_n)^{1/2} \delta_t, \quad (3)$$

where k_t is the tangential stiffness according to the Hertz theory [4], and δ_t is the tangential displacement vector. Equation (3) is only valid if $|\mathbf{F}_t| \leq \mu_s F_n$ (μ_s is the static coefficient of friction). When that condition is broken or a tangential velocity \mathbf{v}_t between the bodies in contact appears, \mathbf{F}_t is formulated according to Coulomb's model as $\mathbf{F}_t = -\mu_d F_n (\mathbf{v}_t / |\mathbf{v}_t|)$, where μ_d is the dynamic coefficient of friction.

3 Results

The previous formulations are used to simulate the impact dynamics of the two examples shown in Fig. 1. The compass-gait biped is composed of two legs of length $l = 1$ m and mass $m = 5$ kg, the hip is a 10-kg point mass, m_H in Fig. 1a. Figure 2a shows the results for the post-impact velocity of the hip, the separation velocity of the trailing foot, and the energy loss at impact as functions of the foot radius R , varying from 0.05 to 1 m. The pre-impact state is defined by $q_4 = 2q_3 = 40^\circ$, and the stance (rear) foot is rolling without slipping before heel-strike. The pre-impact angular velocities are $\dot{q}_3^- = 1$ rad/s and $\dot{q}_4^- = 0$. The results obtained with the two methods are very similar and the curves have the same tendency. There is a clear correlation between the decrement of the hip velocity and the energy loss. It can also be seen that the radius R is a key parameter to reduce energy losses at impact, increasing R thus being advisable to obtain more efficient walking motion.

The model of the subject with crutches is composed of four links coupled by revolute joints. The anthropometric parameters have been taken from the literature for a male subject of 70 kg and 1.75 m height. The total mass of the two crutches is 1.2 kg and their length is 1 m. Figure 2b shows the results for the post-impact velocity

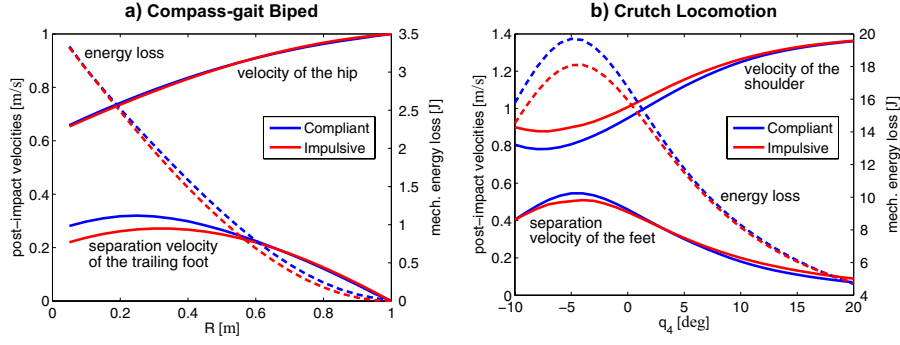


Fig. 2 Results: (a) Compass-gait Biped. Post-impact velocities and mechanical energy loss as functions of the foot radius R . (b) Crutch Locomotion. Post-impact velocities and mechanical energy loss as functions of the hip angle q_4 .

of the shoulder, the separation velocity of the feet, and the energy loss at impact as functions of the hip angle q_4 , varying from -10° to 20° . The pre-impact state is defined by $q_3 = 10^\circ$, $q_5 = 150^\circ$, and q_6 guarantees the crutch-ground contact. The only nonzero pre-impact angular velocity is $\dot{q}_3^- = 1$ rad/s. Again, both contact formulations lead to similar results in terms of post-impact kinematics and energetics. The separation velocity of the feet is maximum when the torso leans slightly backwards with respect to the legs. Conversely, the velocity of the shoulder after impact increases, and the energy loss decreases, when the torso leans forward.

4 Conclusions

Two different approaches for the analysis of impact in biomechanical systems have been presented. In the first one, the impact condition is characterized by impulsive bilateral constraints, whereas in the second, explicit compliant models are used to formulate the generated contact forces. We have compared the performance of both approaches in two impact situations and shown that the results are quite similar and behave similarly to parameter variations. The advantage of impulsive formulations is their simplicity, since forward dynamics can be solved algebraically. However, time evolution of forces during impact is not accessible. Compliant formulations are computationally costly, but the evolution of contact forces can be estimated.

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