SOME ASPECTS OF HEEL STRIKE IMPACT DYNAMICS IN THE STABILITY OF BIPEDAL LOCOMOTION

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ABSTRACT

Heel impact contributes to the stability of bipedal locomotion: the change of trajectory induced at heel strike has a stabilizing effect, and the phase–space volume contraction to which heel strike contributes is a necessary condition for stability.

For a given passive walker, there is a slope limit to obtain stable limit cycles. However, this limit can be surpassed if extra dissipation is introduced. This stabilizing mechanism can be used or be present in bipedal locomotion. In this paper, it is proposed that heel strike can be accommodated to absorb more or less energy, and then that it is used as a regulation or control mechanism in bipedal locomotion.

From a biomechanical point of view, dissipation through heel impact is the most efficient way to lose energy, any other way requires an active participation of muscles. This signifies the importance of the proposed regulation mechanism.

In this work, some of the control possibilities of heel impact are briefly analyzed related to the energetics in order to claim its role and importance as a control mechanism in bipedal locomotion. This study can be valuable in several areas, for example, in control, prosthetics, shoe design, and generally in further understanding of bipedal locomotion.

1 INTRODUCTION

Heel impact is known to be a major contributor to the stability of bipedal locomotion [Hürmüzlü (1987)], [Goswami (1996)], [Garcia (1998)]. On the one hand, there is the strong "stabilizing effect" due to the change of trajectory that the system undergoes at each heel strike, on the other hand, from a dynamical point of view, there is the energy dissipating nature of the heel impact. The associated phase–space volume contraction has been shown to be a necessary condition for stability [Goswami (1988)].

It is known that for a given passive walker, there is a slope limit from which it is impossible to obtain stable limit cycles. However, this limit can be exceeded if extra dissipation is introduced in the system. For example, in [Goswami (1988)] damping at the hip joint was used for this purpose. This stabilizing mechanism can be used for bipedal locomotion: if the slope increases or decreases, or stopping or accelerating is intended, dissipation can be introduced or removed accordingly.

In this work, it is postulated that mechanical energy dissipation occurring at heel impact can be used to control the dynamics of motion to some extent. For example in [Font-Llagunes (2008)], torso and stance leg angle at heel strike have been shown to have an influence on the energy losses at heel strike. From the above perspective, it is clear the importance of the research related to impact dynamics in understanding bipedal locomotion, something already pointed out by Hürmüzlü [Hürmüzlü (1998)]. Taking into account that heel impact is the most efficient way to remove mechanical energy from a biomechanical point of view –any other forms require an active participation of muscles–, the above mentioned interest is even greater.

In this respect, a paradox has been reported in the context of human locomotion, heel strike peak forces do not change in "in vivo" experiments when comparing two different footwear with different cushioning materials with appreciable different stiffnesses [Nigg (1987)]. It is our belief that such effects can partly be related to stability control based on heel impact configuration change described before.

In this work, we study the dissipating capabilities of heel impact as a stabilizing strategy. To this end, a $2D$ human-like walker model based on anthropometric data has been used. The model captures the most relevant dynamics of human locomotion. Based on bibliographic data for human walking, we define a pre-impact reference state and perform an analysis of the heel-impact contribution to dissipation in that state neighborhood. We consider also the effect of blocking some joints. The computation and presentation of the results is based on the formulation for the orthogonal decomposition of the dynamics at heel strike described in [Font-Llagunes (2009)].

Even though the control strategies analyzed are active, since they imply pre-impact actuation to accommodate system state at heel strike or muscular actuation to block a given joint, they all have a very low energy footprint.

This paper is organized as follows:

In Section 2, the model of the human walker used in order to perform the heel impact simulations is described.

In Section 3, the impulsive dynamics equations used in this research are briefly described.

In Section 4, the orthogonal decomposition of the impulsive dynamics equations used to analyze the simulation results is outlined.

In Section 5, some simulations of heel impact with different conditions are presented. They illustrate the proposed control mechanism.

In Section 6, a comprehensible answer to the above mentioned paradox is given. Evidencing the reality and importance of the given control mechanism.

Finally, in Section 7, the conclusions of this work are summarized.

2 HUMAN WALKER MODEL AND PRE-IMPACT CONDITIONS

In this work, we perform $2D$ simulations of the human multibody model depicted in Fig. 1. The model has been made with the help of 3D_MEC symbolic multibody program [Ros (2005)], used as a pre-processor and post-processor, although actual numerical computations are made in MATLAB. The model is based on the anthropometric geometric and inertial data given in [Dumas (2006)]. The total body mass is assumed to be $75\,Kg$. The model contains a $Torso - Pelvis$, the 2 $Thighs$, the 2 $Shanks$ and the 2 $Feet$, linked by the Hip, $Knee$ and $Ankle$ joints. The position depicted in the figure is the reference pre-impact configuration used in this study. It is adjusted from the data of [Murray (1964)] on normal walking, for the average "tall" subject (man).

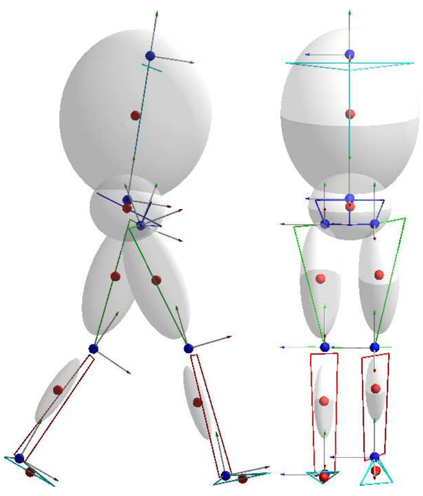

Figure 1: DYNAMIC MODEL OF THE BIPED.

For the simulations, impact is supposed to take place at the *Calcaneous* marker (CAL) of the right foot. The left foot is assumed to contact the ground at the mid point between the $1st$ and $5th$ *Metatarsal Head* markers (MH). The pre-impact velocity of the body segments has been adjusted based on the normal average horizontal velocity given in [Murray (1964)] for the mentioned type of subject. In order to do that, we assume that:

- The $Torso - Pelvis$ does not rotate w.r.t. the inertial frame, and the horizontal velocity component of the *Lumbar Joint Center* (LJC) is given as the average horizontal velocity of [Murray (1964)].
- The point CAL of the right leg has a vanishing horizontal velocity component, and the *Knee* and *Ankle* joints are supposed to have zero relative angular velocity.
- The point MH of the left leg has zero pre-impact velocity, and the *Knee* joint has zero relative angular velocity.

The relative angular velocity of the *Ankle* joint of the left leg, ω_A, can be used as an additional DoF to adjust the initial pre-impact velocity.

3 DYNAMIC EQUATIONS FOR THE IMPULSIVE MOTION AT HEEL IMPACT

Dynamic simulations of impacts can be studied using an impulsive approach. The actual computations are based on the orthogonal decomposition of the dynamics at heel strike described in [Font-Llagunes (2009)]. The heel impact is assumed to be "inelastic", i.e., the colliding point must stay in contact with the ground after the impact. This is a reasonable and widely used assumption in the analysis of walking systems.

The kinetic energy of the walking system can be written as

$$T(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2}\dot{\mathbf{q}}^T \mathbf{M}(\mathbf{q})\dot{\mathbf{q}}, \tag{1}$$

where $\mathbf{M}$ is the symmetric and positive definite mass matrix, and $\mathbf{q}$ and $\dot{\mathbf{q}}$ are the generalized coordinates and velocities of the system, respectively.

The dynamics of this impulsive motion phase can be characterized by impulse-momentum level dynamic equations:

$$\left[\frac{\partial T}{\partial \dot{\mathbf{q}}}\right]_-^+ = \mathbf{M}\left(\dot{\mathbf{q}}^+ - \dot{\mathbf{q}}^-\right) = \bar{\mathbf{f}}_A + \bar{\mathbf{f}}_R, \tag{2}$$

where "$-$" and "$+$" denote the pre- and post-impact instants; $\bar{\mathbf{f}}_A$ and $\bar{\mathbf{f}}_R$ are the impulses of the generalized nonconservative applied forces and the generalized constraint forces; and $[(\partial T)/(\partial \dot{\mathbf{q}})]_-^+ = -\bar{\mathbf{f}}_I$ is the negative of the impulse of the generalized inertial forces. If the applied forces have a non-impulsive nature then $\bar{\mathbf{f}}_A = \mathbf{0}$. This is usually the case of human locomotion since the muscles provide finite forces. When muscles are used to block a given body joint, then the implied forces are considered impulsive constraint forces. For this impulsive motion phase it is also assumed that $\dot{\mathbf{q}}^-$ can be determined based on the previous finite motion analysis of the single support phase.

The constraints in this impulsive motion phase are the contact and non-sliding conditions at the right CAL after impact. These can be expressed as:

$$\mathbf{A}_I\,\dot{\mathbf{q}}^+ = \mathbf{0}. \tag{3}$$

The contact impulses are normally generated by these constraints, hence, $\bar{\mathbf{f}}_R = \mathbf{A}_I^T\bar{\lambda}_I$ where $\bar{\lambda}_I = [\bar{\lambda}_{I_t},\,\bar{\lambda}_{I_n}]^T$ represents the impulse of the constraint forces generated at heel strike.

Should the topology change cause a situation that for left MH gives a velocity with a component facing the ground, then this would mean that the non-colliding stance foot does stay in contact. In this case, Eqn. (3) would include the contact and non-sliding condition of the left foot at MH, as this constraint can also generate impulses. When a joint is blocked during impact –as a heel impact control mechanism– then Eqn. (3) will also contain the corresponding constraint at the velocity level.

4 DECOMPOSITION OF THE IMPULSIVE MOTION

Based on the Jacobian defining the inert constraints in Eqn. (3), the tangent space of the configuration manifold of the walking system can be decomposed to the spaces of *constrained* and *admissible* motions [Kövecses (2003a), Kövecses (2003b)] for the pre-impact instant. This will then also hold for the entire duration of the contact onset, since the configuration of the system does not change during this short period of time. The two subspaces can be defined so that they are orthogonal to each other with respect to the mass metric of the tangent space. This decomposition can be accomplished via two projector operators [Kövecses (2003a), Kövecses (2003b)]. The projector associated with the space of constrained motion can be written as

$$\mathbf{P}_c = \mathbf{M}^{-1}\mathbf{A}_I^T\left(\mathbf{A}_I\mathbf{M}^{-1}\mathbf{A}_I^T\right)^{-1}\mathbf{A}_I, \tag{4}$$

and the projector for the space of admissible motion can be obtained as

$$\mathbf{P}_a = \mathbf{I} - \mathbf{P}_c = \mathbf{I} - \mathbf{M}^{-1}\mathbf{A}_I^T\left(\mathbf{A}_I\mathbf{M}^{-1}\mathbf{A}_I^T\right)^{-1}\mathbf{A}_I, \tag{5}$$

where $\mathbf{I}$ denotes the identity matrix. These projectors are not symmetric, which is a direct consequence of the nature of the metric of the tangent space. Based on them, the generalized velocities of the system can be decomposed as

$$\dot{\mathbf{q}} = \mathbf{v}_c + \mathbf{v}_a = \mathbf{P}_c\dot{\mathbf{q}} + \mathbf{P}_a\dot{\mathbf{q}}, \tag{6}$$

which represent the two components associated with both subspaces. It is interesting to note that in general $\mathbf{v}_c = \mathbf{P}_c\dot{\mathbf{q}}$ and $\mathbf{v}_a = \mathbf{P}_a\dot{\mathbf{q}}$ are non-holonomic components. Any vector of generalized forces or generalized impulses can also be decomposed using the transpose of the operators given above [Kövecses (2003a)]. Then, for the impulsive case we have

$$\bar{\mathbf{f}} = \bar{\mathbf{f}}_c + \bar{\mathbf{f}}_a = \mathbf{P}_c^T\bar{\mathbf{f}} + \mathbf{P}_a^T\bar{\mathbf{f}}. \tag{7}$$

Based on Eqns. (1) and (6), it can be shown that the kinetic energy can also be decomposed as

$$T = T_c + T_a = \frac{1}{2}\mathbf{v}_c^T\mathbf{M}\mathbf{v}_c + \frac{1}{2}\mathbf{v}_a^T\mathbf{M}\mathbf{v}_a. \tag{8}$$

To obtain this equation it was used that the projectors in (4) and (5) are orthogonal with respect to the system mass metric, i.e., $\mathbf{P}_c^T\mathbf{M}\mathbf{P}_a = \mathbf{P}_a^T\mathbf{M}\mathbf{P}_c = \mathbf{0}$. Therefore, $\mathbf{v}_c^T\mathbf{M}\mathbf{v}_a = \mathbf{v}_a^T\mathbf{M}\mathbf{v}_c = 0$. Any force or impulse arising in the space of constrained motion will change only T_c, leaving T_a unaffected and vice versa [Modarres (2008)]. The impact characterized by the constraints in (3) gives rise to impulses which will influence quantities in the space of constrained motion only.

Based on the above decompositions, it can be seen that the impulse-momentum level dynamic equations in (2) can be decoupled as

$$\left[\frac{\partial T_c}{\partial \mathbf{v}_c}\right]_-^+ = \mathbf{M}\left(\mathbf{v}_c^+ - \mathbf{v}_c^-\right) = \mathbf{A}_I^T\bar{\lambda}_I, \tag{9}$$

which are the impulse-momentum level dynamic equations for the space of constrained motion, and

$$\left[\frac{\partial T_a}{\partial \mathbf{v}_a}\right]_-^+ = \mathbf{M}\left(\mathbf{v}_a^+ - \mathbf{v}_a^-\right) = \mathbf{0}, \tag{10}$$

which describes the impulsive dynamics associated with the space of admissible motion. From Eqn. (10) and using that $\mathbf{M}$ is positive definite, it is immediately visible that $\mathbf{v}_a^+ = \mathbf{v}_a^-$. Based on Eqns. (3) and (6) it can also be concluded that $\mathbf{v}_c^+ = \mathbf{0}$. Considering this and using Eqn. (9), we can write that

$$-\mathbf{M}\mathbf{v}_c^- = \mathbf{A}_I^T\bar{\lambda}_I, \tag{11}$$

and also, taking into account the velocity decomposition in (6), we obtain the following expression to solve for the post-impact generalized velocities

$$\dot{\mathbf{q}}^+ = \mathbf{v}_a^+ = \mathbf{v}_a^- = \mathbf{P}_a\dot{\mathbf{q}}^-. \tag{12}$$

Based on Eqns. (4), (6) and (11) we can also obtain the solution for the generalized constraint impulses as

$$\bar{\lambda}_I = -\left(\mathbf{A}_I\mathbf{M}^{-1}\mathbf{A}_I^T\right)^{-1}\mathbf{A}_I\dot{\mathbf{q}}^-, \tag{13}$$

which appear in order to set the velocity of the colliding foot to zero. The normal component of $\bar{\lambda}_I$ is the impulse perpendicular to the ground and it is usually associated with the deformation of the colliding bodies (foot-ground) in this direction. The other component is the impulse in the tangent direction which is more complex in nature, since it can come either from friction or from tangential compliance of the colliding bodies. If the left foot generates impulsive forces, or some of the joints of the system are blocked, as a heel impact controlling mechanism, the associated impulses will accordingly obtained, from Eqn. (13).

We define the following index to quantify the energetic aspects heel impact controlling mechanism:

$$\xi_I = \frac{T_c^-}{T^-} = \frac{\left(\dot{\mathbf{q}}^-\right)^T \mathbf{P}_c^T \mathbf{M} \mathbf{P}_c \dot{\mathbf{q}}^-}{\left(\dot{\mathbf{q}}^-\right)^T \mathbf{M} \dot{\mathbf{q}}^-}, \tag{14}$$

where $\xi_I \leq 1$ (non-dimensional) represents the ratio of the total pre-impact kinetic energy T^- that is lost at heel strike, i.e., the local energetic efficiency of the impact.

5 RESULTS AND DISCUSSION

In this section, the decomposition of the kinetic energy at the pre-impact time of the heel strike event is used to analyze possible heel strike control strategies. This analysis does not pretend to be exhaustive, it is just used to support our thesis, i.e., the idea that the heel strike can be used as a regulation mechanism in human locomotion.

In what follows, simulations of three representative examples are presented:

1. Pre-impact left ankle joint velocity change: This change can influence normal impact velocity and then the amount of energy removed at heel strike.

2. During-impact right ankle joint blocking: The enforced lack of compliance of the ankle joint will make the impact much more stronger.

3. Pre-impact $Torso - Pelvis$ orientation change effect: This allows to change the inertia distribution and then it should affect the energetics of impact.

These examples cover a wide conceptual spectrum: Actuation strategies at the pre-impact configuration and velocity levels, and muscle actuation to constrain motion. They are enough to get a grasp of the different actuation possibilities. Nevertheless, other actuations different from the ones exemplified in the following can possibly exhibit bigger possibilities, and they can, therefore, be more appealing. Further work will be done in this direction.

Example 1

Two simulations are presented in Fig. 2, no joint is blocked, the initial (pre-impact) condition differs only in velocities: In the first simulation (first row of Fig. 2) the initial velocity is specified with left ankle joint velocity $\omega_A = +1 rad/s$. In the second simulation the left ankle joint velocity is $\omega_A = -1 rad/s$ (second row of Fig. 2). The green arrows indicate the initial velocities, and the blue and red arrows show the admissible and constrained velocities, respectively.

In the first simulation, the velocity normal to the ground of point CAL of the right leg is smaller. This implies a lower value of the energy dissipated at heel strike. The figure of merit for energy dissipation obtained for these simulations are:

1. $\xi_I = T_c/T = 0.011$
2. $\xi_I = T_c/T = 0.017$

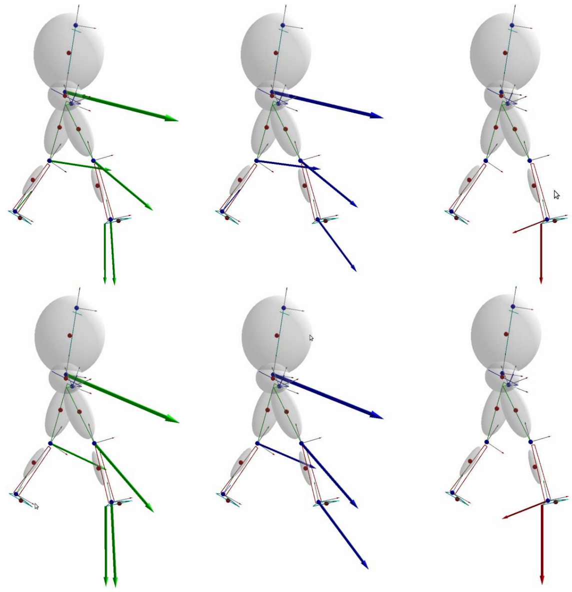

Figure 2: Example 1.

Example 2

Two simulations are presented in Fig. 3, the initial conditions are exactly the same: The first simulation (first row of Fig. 3) is the same that the first one in Example 1. In the second simulation the right ankle joint is blocked (second row of Fig. 3). As before, green arrows indicate the initial velocities; and blue and red arrows show the admissible and constrained velocities, respectively.

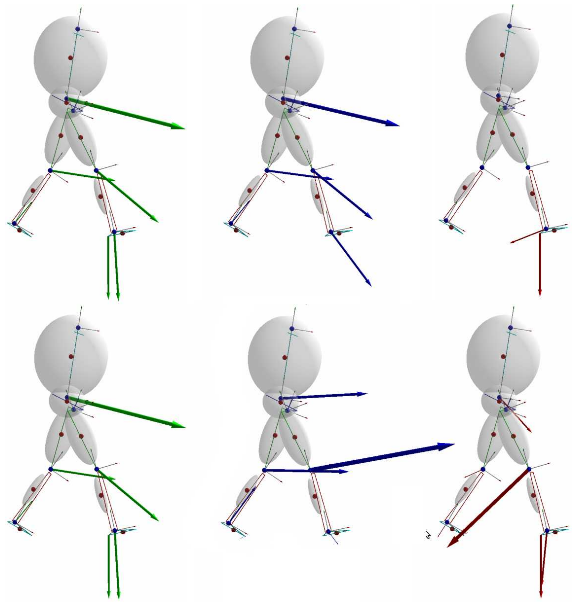

Figure 3: Example 2.

The figure of merit for energy dissipation obtained for these simulations are:

1. $\xi_I = T_c/T = 0.011$

2. $\xi_I = T_c/T = 0.181$

These results reveal that ankle joint blocking is a very efficient mechanism to regulate the energy dissipated by the heel strike. Almost 20% of the kinetic energy of the system can be absorbed by blocking the ankle. Obviusly, from a non-impulsive framework point of view, this implies that we can play with ankle blocking level to manage a huge amount of the kinetic energy, that can be removed –if desired– with almost no aditional energy expenditure.

Example 3

Two simulations are presented in Fig. 4, initial conditions are equivalent in terms of velocities, right ankle joint is blocked in both, only torso position changes: In the first simulation (first row of Fig. 4) the torso position is as in previous simulations. In the second one (second row of Fig. 4) the torso is inclined backwards 10^o. As before, green arrows indicate initial condition; blue and red arrows admissible and constrained velocities, respectively. The figure of merit for energy dissipation are:

1. $\xi_I = T_c/T = 0.181$

2. $\xi_I = T_c/T = 0.190$

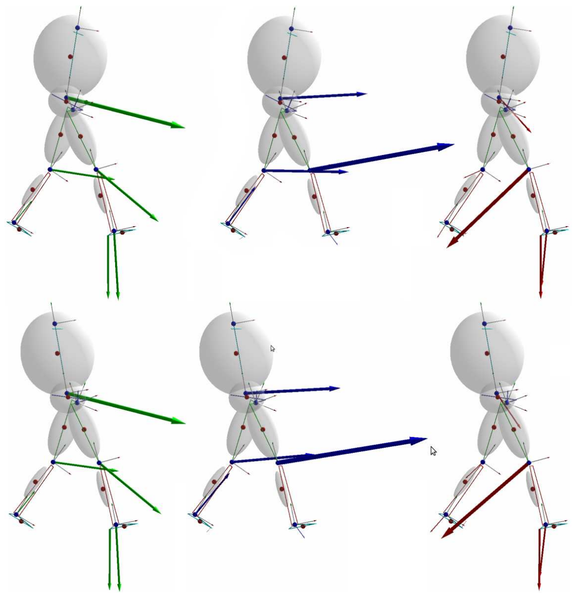

Figure 4: Example 3.

It is shown that the torso position has an effect on energy dissipation. In fact, leaning the torso backwards increases the energetic losses which is in agreement with the results obtained in [Font-Llagunes (2008)] using a simpler model. Note that however, for the considered angles, the most important contribution to energy dissipation is given by the right ankle joint blocking.

6 FURTHER EVIDENCE

A paradox has been reported in the context of human locomotion, heel strike peak forces do not change in “in vivo” experiments when comparing two different footwear with different cushioning materials with appreciable different stiffnesses [Nigg (1987)]. For similar experiments with cadavers, an increase of peak forces is observed when cushioning material stiffness increases.

We think that the answer to this paradox, to a great extent, can be given based in the analysis presented previously:

Differences in stiffnesses and damping of cushioning material at the heel influence heel strike forces –as experiments with passive subjects demonstrate– and as a consequence they also influence energy dissipation.

An erroneous design hypothesis supposes that changing the footwear characteristics will not essentially change the kinematics of gait and, as a consequence, that the effect of a softer cushioning material would be to lower the peak forces at heel strike.

The strong role of heel impact on overall walking stability, implies that changing the impact levels affects stability. This is because mechanical energy dissipation level would change if gait does not change. If no change on the gait is done a lower level of dissipation will make the system less stable. Note that a very compliant material with low damping, can bounce back part of the impact energy, lowering the overall dissipation due to heel strike if gait does not change.

Therefore, the brain needs to act to adapt the gait to the new scenario. The previous analysis suggests that heel strike accommodation can easily and efficiently compensate for the changes in the mechanical energy dissipation, and then to increase the stability of the system. For example, from the simulations it is obvious that this can be accomplished using different levels of joint ankle rigidization.

This explains how a softer sole does not lowers the heel strike forces as much as expected, as heel strike is accommodated to dissipate more energy, what in turns compensates the expected heel strike force decrease.

The above discussion strongly suggests the presence of the proposed regulating mechanism in real human locomotion.

The proposed control strategy seems to be equally applicable in the context of control of robotic bipedal walkers. For instance model predictive control methods can be used to straightforwardly implement such a control strategy.

7 CONCLUSIONS

We have started this paper reminding the acknowledged role that the existing bibliography gives to mechanical energy dissipation at heel impact on bipedal locomotion stability. Based on this, we postulate that the heel strike can be accommodated in different ways to produce different levels of mechanical energy dissipation, and that this can be used as a regulating effect in the control of bipedal locomotion. We explain that this mechanism can be used to increase limit cycle stability when adapting to a changing slope or changing locomotion velocity. Based on biomechanical principles, we also conclude that –within its physical limits–, heel strike is by far be the most efficient way to dissipate excess mechanical energy to stabilize locomotion.

A set of illustrative examples are used to show that there are different ways in which heel impact can accommodate different levels of energy dissipation. Although the objective of this work was not to do an exhaustive and systematic research on the different control strategies for energy dissipation at heel impact, we have found easy-to-implement control mechanisms that can greatly affect energy dissipation at heel impact, as for example ankle joint rigidization.

Further evidence of the actual use of such a heel strike adaption as a control mechanism for bipedal locomotion is based on the explanation of a well know paradox. This, fact reveals the importance of the proposed regulating mechanism, and the need to further investigate it, for the fields of prosthetics, shoe design, control of bipedal locomotion systems and generally in further understanding of bipedal locomotion.

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