The Bristle Friction Model in a Linear Complementarity Formulation for Multibody Systems with Contacts

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The simulation of multibody systems with contacts presents some well-known challenges, especially when it involves frictional contacts. Moreover, dealing with large systems and many contacts makes it much harder to achieve good simulation performance. That is why some authors have proposed several formulations with the form of *Linear Complementarity Problems* (LCP). For example, Stewart and Trinkle [1] proposed a position-level integration scheme for multibody systems with frictional contacts, while Anitescu and Potra [2] formulated it at the velocity-level. In both cases, a linear approximation of the Coulomb friction model was used.

With the aim of improving the numerical behaviour of the Coulomb model due to non-smoothness, several authors [3, 4] have proposed friction models using the *bristle* approach, which models the surface asperities with flexible elements. Essentially, this approach defines the stiction force in terms of the average *bristle deflection* $\mathbf{s}$ as

$$\mathbf{F}_{\text{stic}} = -k_b \mathbf{s} \tag{1}$$

where k_b is the effective bristle stiffness, and the sliding velocity $\mathbf{v}_T$ is integrated to compute the bristle deflection as $\mathbf{s} = \int \mathbf{v}_T dt$. However, other phenomena might also be taken into account, such as bristle damping, viscous friction, or the Stribeck and dwell-time effects.

The complementarity formulation proposed here consists in a velocity-level representation for multibody systems with contacts and other constraints, where a *bristle* friction model is used. Let $\mathbf{q}$ be the $p \times 1$ array of generalized coordinates of a multibody system. A set of n generalized velocities $\mathbf{v}$ is defined, such that $\dot{\mathbf{q}} = \mathbf{\Gamma}\mathbf{v}$, where $\mathbf{\Gamma}(\mathbf{q},t)$ is the $p \times n$ transformation matrix. With this representation of the system, the dynamic equations are

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{c} = \mathbf{f}_{\text{app}} + \mathbf{f}_{\text{bilat}} + \mathbf{f}_{\text{cont}} + \mathbf{f}_{\text{fric}} \tag{2}$$

where $\mathbf{M}(\mathbf{q})$ is the $n \times n$ mass matrix, $\mathbf{c}(\mathbf{q},\mathbf{v})$ is the $n \times 1$ array containing the Coriolis and centrifugal terms, $\mathbf{f}_{\text{app}}$ is the generalized applied force, $\mathbf{f}_{\text{bilat}}$ and $\mathbf{f}_{\text{cont}}$ are the generalized bilateral and unilateral constraint forces, respectively, and $\mathbf{f}_{\text{fric}}$ is the generalized friction force.

The m bilateral constraints (holonomic and non-holnomic) are defined at the velocity level as $\mathbf{A}\mathbf{v} = \mathbf{0}$, where $\mathbf{A}(\mathbf{q})$ is the $m \times n$ constraint Jacobian matrix. For the r contacts, i.e., unilateral constraints, the *gap functions* that define the distance between contacting surfaces are arranged into the array $\mathbf{\Phi}(\mathbf{q}) \geq \mathbf{0}$. Then, the normal serparation velocity is $\dot{\mathbf{\Phi}} = \mathbf{N}\mathbf{v} \geq \mathbf{0}$, where $\mathbf{N}(\mathbf{q})$ is the $r \times n$ contact Jacobian matrix.

With the above kinematic constraint equations, the generalized forces for the bilateral constraints and the contacts are defined as $\mathbf{f}_{\text{bilat}} = \mathbf{A}^{\text{T}}\boldsymbol{\lambda}$ and $\mathbf{f}_{\text{cont}} = \mathbf{N}^{\text{T}}\boldsymbol{\lambda}_N$, where $\boldsymbol{\lambda}$ contains the m multipliers of the bilateral constraints, and $\boldsymbol{\lambda}_N$ contains the r normal contact forces. Due to its unilateral nature, the normal force cannot be negative, and have to be zero when separation occurs, so the complementarity condition is defined between $\boldsymbol{\lambda}_N$ and $\dot{\mathbf{\Phi}}$.

The tangent plane of each contact point is characterized using two orthogonal directions, in which the components of the sliding velocity at the contact points can be expressed as $\mathbf{v}_T = \mathbf{D}\mathbf{v}$, where $\mathbf{D}(\mathbf{q})$ is the $2r \times n$ friction jacobian matrix. Therefore, the generalized friction force is $\mathbf{f}_{\text{fric}} = \mathbf{D}^{\text{T}}\boldsymbol{\lambda}_T$, where $\boldsymbol{\lambda}_T$ is the array that contains the $2r$ components of the friction forces.

In order to calculate the friction force, the sliding velocity is integrated explicitly to get the average bristle deflection of the next instant $\mathbf{s}^+ = \mathbf{s}^- + h\mathbf{v}_T^+$, where the superscripts $^+$ and $^-$ denote next and current instant, respectively, and h is the time-step size. Then, the friction force array becomes

$$\boldsymbol{\lambda}_T^+ = -\mathbf{K}_T\mathbf{s}^+ = \boldsymbol{\lambda}_T^- - \mathbf{K}_T h\mathbf{D}\mathbf{v}^+ \tag{3}$$

where $\mathbf{K}_T$ is a $2r \times 2r$ diagonal matrix containing the bristle stiffnesses, and $\boldsymbol{\lambda}_T^- = -\mathbf{K}_T \mathbf{s}^-$ is the friction force of the previous instant. The above expression shows that the average bristle deflection is no longer necessary to calculate the friction force, because all the needed information is contained in the previous friction force $\boldsymbol{\lambda}_T^-$.

In the bristle model, the boundary of the static friction force is the same as the Coulomb model, which imposes a limit to the magnitude of the friction force vector. However, an approximation is made here, and the limits are enforced component-wise; so that the dynamic equations can have a linear form. Then, the lower and upper bounds for the two components of the friction force at the i-th contact point are $\underline{\lambda}_{Ti} = -\mu_i \lambda_{Ni}^-$ and $\overline{\lambda}_{Ti} = +\mu_i \lambda_{Ni}^-$, respectively, where λ_{Ni}^- is the normal force at the previous instant. Nevertheless, a few iterations can be performed in order to make the boundaries tend to $\pm\mu\lambda_{Ni}^+$.

The velocity-level formulation is achieved by means of a first order discretization, so that the generalized velocities of the next instant are $\mathbf{v}^+ = \mathbf{v}^- + h\dot{\mathbf{v}}^+$. This expression is introduced into Eq. (2), and together with the constraint equations and the expression for the friction force components in Eq. (3), they can be expressed in the following from

$$\begin{bmatrix} \mathbf{M} & -\mathbf{A}^{\mathrm{T}} & -\mathbf{N}^{\mathrm{T}} & -\mathbf{D}^{\mathrm{T}} \\ \mathbf{A} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{N} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{D} & \mathbf{0} & \mathbf{0} & \tilde{\mathbf{C}}_T \end{bmatrix} \begin{Bmatrix} \mathbf{v}^+ \\ h\boldsymbol{\lambda}^+ \\ h\boldsymbol{\lambda}_N^+ \\ h\boldsymbol{\lambda}_T^+ \end{Bmatrix} + \begin{Bmatrix} -\mathbf{M}\mathbf{v}^- - h\left(\mathbf{f}_{\mathrm{app}} - \mathbf{c}\right) \\ \mathbf{0} \\ \mathbf{0} \\ -\tilde{\mathbf{C}}_T h\boldsymbol{\lambda}_T^- \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \\ \dot{\boldsymbol{\Phi}}^+ \\ \boldsymbol{\sigma}^+ \end{Bmatrix} \tag{4}$$

where $\tilde{\mathbf{C}}_T = \left(\mathbf{K}_T h^2\right)^{-1}$, and $\dot{\boldsymbol{\Phi}}^+$ and $\boldsymbol{\sigma}^+$ are the slack variables for $\boldsymbol{\lambda}_N^+ \in [0, +\infty)$ and $\boldsymbol{\lambda}_T^+ \in \left[\underline{\boldsymbol{\lambda}}_T, \overline{\boldsymbol{\lambda}}_T\right]$ respectively. This form is known as *Mixed Linear Complementarity Problem with Bounds*, which is the general version of the LCP, where an interval is defined for each variable.

Simulations have been carried out with the model of a wheel (Fig. 1 left) with one contact point with the ground, and the initial conditions make it slide for 2 seconds. The results have been compared with the bristle model proposed by Liang et al. [4], which is the non-linear version of the formulation presented here.

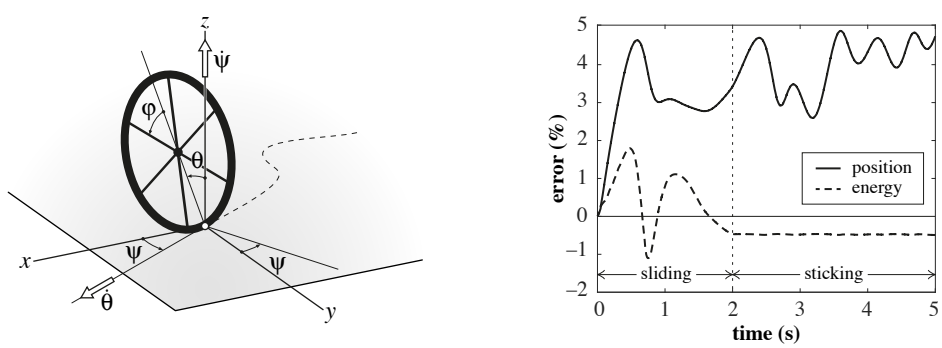

Fig. 1: Model of the wheel on the ground (left) and error of the simulation compared with the Liang et al. [4] (right).

This formulation can be easily enhanced by relaxing the constraints to make it able to cope with redundancy. Nevertheless, the bristle model on its own allows the system to handle redundancy coming from the friction. Though a velocity regularization of the stiction force solves the same problem, the sliding artifact does not appear when using the bristle model.

References

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